

Model for Early Detection of Emergency Conditions in Power Plant Equipment Based on Machine Learning Methods

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Received June 19, 2018; revised August 21, 2018; accepted August 29, 2018

Abstract—The article discusses a method for early detection and prediction of abnormality in operation of power-unit process equipment taking as an example the PTN 1100-350-17-4 turbine driven feedwater pump of a 300 MW power unit. The importance of the problem of predicting possible process equipment malfunctions at an early state of their occurrence is determined, and the specific features of solving it in the power industry are explained. The range of process equipment defects that can be efficiently detected using the predictive analytics methods is outlined. The fundamental assertion stating that the scope of analog and discrete measurements available in the process control system's set of computerized automation tools is sufficient for applying the predictive analytics methods is emphasized. Modern predictive analytics methods are briefly reviewed, and the specific features of model training algorithms are mentioned. Separate attention is paid to the problems of preparing initial data for training the model. The mathematical problem of modeling an abnormality indicator taking the values from 0 (normal operation) to 1 (abnormal operation) is formulated. In turn, this problem is formulated as the binary classification problem of attribute vectors characterizing the equipment state at the given moment of time. An original approach is suggested, which combines the multivariate state estimation technique (MSET), in which the degree of abnormality in a technical state is determined from the extent to which the Hotelling criterion exceeds a threshold level (which is automatically calculated in the algorithm), and machine learning methods, the use of which makes it possible to overcome a number of difficulties inherent in the MSET. For solving the problem of determining the composition of the most informative attributes from the values of which early development of an emergency can be detected, it is proposed to use an ensemble of regression models. A method for selecting the modeled variable and the set of regressors is substantiated. An abnormality indicator calculation method based on composing an ensemble of linear regression models is proposed, and the advantage of using an ensemble over a single classifier is shown. A method for producing an alarm in response to detected abnormality in the operation of power unit process equipment is suggested. It is shown that it became possible by using the proposed model to detect the onset of the emergency development process, whereas individual indicators failed to reveal pump operation singularities in the preemergency interval of time.

Keywords: process equipment, detection of abnormalities, predictive analytics, committee of classifiers, logistic regression

DOI: 10.1134/S0040601519030042

In the period of actively using process equipment, events (accidents) that may adversely affect it or cause its failure inevitably occur. A model able to predict the future emergency would make it possible to timely take measures for eliminating it, thus helping to achieve more efficient use of process equipment. Development and investigation of such models is the subject of predictive analytics [1, 2].

Against the background of the predictive analytics methods being widely used in various areas of human activity (financial services, insurance, telecommunications, trade, health care, etc.), the advances of predictive analytics in the power industry look rather modest. There is a simple explanation for this fact. For almost

the century-long history of the power industry in Russia, power engineers have defined the scope of measured parameters characterizing the state of power equipment that allows failure-free operation of the equipment to be maintained subject to fulfilling all necessary conditions (operational regulations). In the overwhelming majority of cases, measures that must be taken to prevent a malfunction or an accident from occurring do not involve the need of using any special methods but are carried out in response to certain recorded measurements (when warning or emergency alarms come in action). The occurring accidents are primarily due to failure to follow the equipment operation rules and regulations or due to external influencing

factors (accidents in the power grid, natural phenomena, human factor, etc.), which cannot be foreseen.

Nonetheless, it can be assumed with a large degree of confidence that there are defects that lead to an accident unless if they are timely removed. The incipience and development of such defects is not revealed by means of individual measurements, but they can be diagnosed using the predictive analytics methods from the behavior of a certain totality of measured parameters [3]. In this connection, there is a need to develop mathematical models able to predict the occurrence of emergencies in advance.

It is important to note that the application of predictive analytics methods for power unit equipment does not involve the need to install new sensors (the scope of necessary measurements is specified by equipment manufacturers, power unit designers, and power plant process engineers).

The basic idea of predictive analytics is that the occurrence of an accident can be predicted with certain probability by continuously analyzing the data characterizing the performance of the equipment being monitored. The prediction can be regarded as effective if it was done a few days before the accident.

The modern trends in predictive analytics combine the methods of statistical and intellectual analysis of data with the use of learned algorithms [4]. They are inherent in all of the presently available predictive analytics methods applied in thermal power engineering and imply preliminary “teaching” of the model on the basis of available input data. Such input data include “historical” values of measured parameters characterizing the operation of particular process equipment; these data are taken from the archives stored in the power unit process control system’s set of computerized automation tools for a long time of power unit operation (usually 1–3 years). In addition, data about the defects (malfunctions) revealed for this period of time, which can lead to an accident unless having been removed, are also used. The equipment operation periods of time with and without such defects are called abnormal and normal, respectively.

Learning algorithms are determined by the predictive analytics methods. For example, the regression model method uses learning intervals for establishing the regression coefficients and the calculated criterion threshold (the model output parameter) classifying the equipment operation time (normal/abnormal). The method of artificial neural networks tunes the weighing coefficients of neurons on learning intervals.

It should be noted that adequate data on the defects of diagnosed equipment are of key importance for elaborating a high-quality predictive model. Unfortunately, the experience we have gained in dealing with the input data on the power units from different power plants shows that almost all defect logs have the same essential deficiencies: not all defects are recorded, and their dates correspond, in the best case, to the time of

their revealing (and not the time of their occurrence). As regards the dates at which the revealed defects were removed, such information is often lacking.

Some of the modern computerized automatic control toolsets installed at power plants may include built-in systems performing early failure diagnostic functions. These systems are, as a rule, based on statistical abnormality detection models operating according to the following principle: if the equipment current state differs essentially from its state typical for normal operation mode, this is a sign of abnormal operation [5]. The drawbacks of these systems are, on the one hand, that they frequently detect an abnormality too late when there is already no time for removing it and, on the other hand, that they produce a significant number of false warnings about a possible accident.

In this article, we propose a method for detecting abnormalities in plant operation that uses the idea of the MSET technique [6] jointly with using the machine learning models by means of linear and logistic regression¹.

STATEMENT OF THE PROBLEM

The plant operation at each moment of time t is described by the vector of indicators $x(t) = [x_1(t), \dots, x_m(t)]^T$, where m is the number of indicators and T is the number of accomplished measurements. In carrying out its measurements by means of a monitoring system at a certain step Δt (e.g., 5 min), a sequence of vectors united into the matrix $X = [x(1), \dots, x(T)]$ is obtained.

Each moment of time t , $t = \overline{1, T}$ is related by an expert to one of two classes, one of which corresponds to normal operation, and the other to abnormal (preemergency or emergency) state of the plant. Denoting the class tag at the time moment t as $y(t)$ ($y(t) = 0$ for the normal state and $y(t) = 1$ for the emergency state), we write the vector of tags as follows: $y = [y(1), \dots, y(T)]$.

Now, to develop a model from the composition of available measurements, it is necessary to determine the output variable and the input variables that “explain” it. Models developed according to the MSET technique are related to the class of “self-associative” ones, in which the sets of input and output parameters used to construct a regression model in the plant normal operation mode coincide with each other [6, 7]. Such models are especially suitable under the physical conditions in which the observed parameters, on the one hand, are closely interconnected and, on the other hand, it is difficult or unreasonable to sepa-

¹ The MSET abbreviation stands for the multivariate state estimation technique. According to the MSET technique, the degree of abnormality in a technical state is determined by the extent to which the Hotelling criterion exceeds a threshold level automatically calculated by the algorithm.

rate explaining and explained parameters from them as is done in constructing cause-and-effect models. The composition of these variables determines in many respects the predictive properties of model error deviations from the observed values. It is reasonable to assume that the model's output variables depend not only on the observed plant indicators but also on derivative indicators, which are not explicitly measured (e.g., relative quantities, derivative quantities, etc.). The problem of shaping a new extended composition of indicators z_1, \dots, z_M based on the initial indicators x_1, \dots, x_m involves the need to engage experts and to understand which particular derivative indicators could be sensitive to development of an accident.

Another problem relating to the MSET technique is connected with calculating the degree to which the plant operation deviates from its normal mode. In the classic version of the MSET technique [6], the decision about abnormality is made based on the results of comparing the outputs obtained from the constructed regression model with the modeled quantity's observed values. In practice, high mismatches do not always mean that there is an accident, and vice versa, some types of preemergency situations do not always manifest themselves in a growth of mismatches.

The task we set forth in this study is to construct ensembles of regression models [8, 9] that use different compositions of input and output variables and models for calculating the abnormality indicator. Each model within an ensemble calculates its own plant operation abnormality indicator $p_i(t)$ at the time moment t for $t = \overline{1, T}$, $i = \overline{1, N}$, where N is the number of models in the ensemble. The final decision $p(t)$ is made in accordance with the ensemble decision rule.

At each moment of time, the abnormality indicator $p(t)$ takes a value from the interval (0; 1). Values close to 0 correspond to normal operation of the plant, and values close to 1 correspond to its abnormal operation.

ABNORMALITY INDICATOR CALCULATION METHOD

The proposed algorithm for calculating the plant operation abnormality indicator includes the following steps.

1. The attributes z_1, \dots, z_M are composed proceeding from the observed plant operation indicators x_1, \dots, x_m , and their values $z(t) = [z_1(t), \dots, z_M(t)]^T$ at each moment of time t , $t = \overline{1, T}$ are calculated. As a result, a sequence of vectors united into the matrix $Z = [z(1), \dots, z(T)]$ is obtained. Thus, the indicator values matrix Z of the dimension $M \times T$ and the vector of tags y of the dimension T constitute the input data array D for constructing the predictive model.

In this study, we consider the PTN 1100-350-17-4 turbine driven feedwater pump (TFWP) of a 300-MW power unit as the monitored equipment set under study.

The following composition of pump indicators $z_1(t), \dots, z_M(t)$ at the time moment t was selected proceeding from the opinions of experts:

(i) the observed values $x_1(t), \dots, x_m(t)$ (e.g., lubrication oil temperature and pressure, turbine bearings vibration velocities, steam temperature and pressure at the TFWP inlet and outlet, etc.); and

(ii) the values $x_1(t), \dots, x_m(t)$ normalized for the feedwater flowrate $x^*(t)$, i.e., $\frac{x_1(t)}{x^*(t)}, \dots, \frac{x_m(t)}{x^*(t)}$, in the given case, $M = 2m$.

2. The initial sampling D is decomposed into non-overlapping sets: two training samples D_{tr1}, D_{tr2} and one test sample D_{tst} , $D = D_{tr1} \cup D_{tr2} \cup D_{tst}$, which are selected in a random manner in a specified ratio. The sets of the corresponding moments of time are denoted by T_{tr1}, T_{tr2} and T_{tst} , $T_{tr1} \cup T_{tr2} \cup T_{tst} = \{1, \dots, T\}$. The training sample D_{tr1} is used to construct regression models; the sample D_{tr2} is used to construct models for calculating the plant operation abnormality indicator, and the sample D_{tst} is used for finally testing the model.

3. At this step, K linear regression models are constructed from the training sample D_{tr1} data, and the model errors are calculated using the sample D_{tr2} data. Let z_{i_k} and ζ_k be the output (modeled) variable and the set of model inputs (regressors), respectively, for the k th regression model, $i_k \in \{1, \dots, M\}$, and ζ_k is a certain subset of attributes $\{z_1, \dots, z_M\}$ that does not contain the modeled attribute z_{i_k} , $k = \overline{1, K}$. Then, the k th regression model will have the following form:

$$z_{i_k} = \varphi_k(\zeta_k) + e_k, \quad k = \overline{1, K},$$

where φ_k is the regression function linear with respect to the attributes ζ_k , and e_k is the model error.

The modeled attribute z_{i_k} and the composition of regressors ζ_k are selected for each regression model in a random manner. For example, for one model, the steam pressure at the TFWP outlet was adopted as the output variable, and the bearing horizontal and vertical vibration velocities and the steam temperature and pressure at the TFWP inlet were taken as the input variables. For another model, the first bearing's horizontal vibration velocity was taken as the output variable, and the temperatures of turbine thrust bearings, the lubrication oil pressure, and steam temperature at the TFWP inlet and outlet were taken as the input variables.

Owing to random selection of the modeled variable and the set of regressors, it becomes possible to avoid the need to engage experts in constructing the regression models at this stage. At the same time, if the composition of these variables has been selected unfortu-

nately, i.e., exact modeling of the output variable is impossible, such models are rejected, and the composition of variables is played out again. The model accuracy is estimated from the determination coefficient on the training sample D_{tr1} .

The parameters of linear regression functions $\varphi_1, \dots, \varphi_K$ were determined using the least squares method [10].

We introduce the following notation: $e(t) = [e_1(t), \dots, e_K(t)]^T$, where $e_k(t) = z_{ik}(t) - \varphi_k[\zeta_k(t)]$ are the vectors of deviations between the regression model outputs and the observed values at the time moment t , $t \in T_{tr2}$. As a result, we obtain the deviation matrix E of the dimension $K \times T_{tr2}$ composed of the vectors $e(t)$, $t \in T_{tr2}$.

4. The matrix E data and the corresponding tags of classes at the time moments t , $t \in T_{tr2}$ are used to train the ensemble of N binary classifiers represented by the selected logistic regressions². Each binary classifier operates in its own space of attributes, which consists of randomly selected regression remainders obtained at the previous step. Thus, each binary classifier solves its own classification problem; i.e., it relates the vector of regression remainders arriving at its input to one of two classes: normal or abnormal operation. For example, the input of the first classifier receives the regression remainders obtained in modeling steam pressure at the TFWP outlet and the horizontal vibration velocity of the first bearing, whereas the input of the second classifier receives the regression remainders obtained in modeling the temperature of turbine thrust bearings, steam temperature at the TFWP inlet and outlet, etc.

It has experimentally been shown that individual classifiers provide insufficient accuracies. It is not possible to say—based solely on the values of regression remainders arriving to the input—whether a system is in the normal or emergency (preaccident) mode of its operation. Nonetheless, the accuracies of individual classifiers are better than they are during random classification, which makes it possible to unite them into an ensemble of classifiers. In [11], it is shown that, under certain conditions, the ensemble operation accuracy outperforms the accuracies shown by the classifiers it consists of (weak classifiers).

The mathematical model of the i th weak classifier is given by

$$p_i = \frac{1}{1 + \exp[-\psi_i(\varepsilon_i)]}, \quad i = \overline{1, N},$$

where p_i is the output of the i th weak classifier, $p_i \in (0; 1)$; ψ_i is the regression function linear with

² The term “classifier” is understood to mean an algorithm that tries to predict, proceeding from the data known to it, which of the predetermined classes the new data will be related to. A weak classifier performs classification with an error probability lower than in the case of mere guessing (0.5 for binary classification).

respect to the attributes ε_i ; and ε_i is a subset of the set of attributes $\{e_1, \dots, e_K\}$.

The composition of regressors ε_i for each weak classifier was selected in a random manner. With such selection, there is no need to engage expert knowledge, and it is possible to form a set of weak classifiers for uniting them into a committee. The advantage of a committee of classifiers over a single classifier the input of which would receive all of the obtained remainders e_1, \dots, e_K is that it is more robust, a feature due to which it can be tuned less exactly for a particular set of input attributes, which were also obtained as a result of random selection of regressors at step 3.

If the space of input variables for a weak classifier was selected unsuitably; i.e., the manner in which it separates data belonging to two classes is almost the same as with random separation, such weak classifiers are rejected, and the composition of input variables is played out anew. The values of the AUC ROC indicator on the training sampling D_{tr2} were used as the classes separation measure [12].

The weak classifier's outputs vector at the time moment t is denoted as $p(t) = [p_1(t), \dots, p_N(t)]^T$, where $p_i(t)$ is the output of the i th weak classifier at the time moment t , $t \in T_{tr2}$. As a result, the abnormality indicators matrix P with the dimension NT_{tr2} composed of the vectors $p(t)$, $t \in T_{tr2}$ is obtained.

5. The matrix P data and the corresponding tags at the time moments t , $t \in T_{tr2}$ are used to construct the ensemble Φ decision rule:

$$p = \Phi(p_1, \dots, p_N),$$

where p is the resulting abnormality indicator.

In the simplest case, simple averaging of the weak classifiers' outputs can be used as the decision rule. However, since individual weak classifiers may introduce different contributions in the final decision, and since they may have different accuracy indicators and generalizing abilities, the following logistic regression model is used as the decision rule Φ :

$$\Phi(p_1, \dots, p_N) = \frac{1}{1 + \exp[-(\beta_0 + \beta_1 p_1 + \dots + \beta_N p_N)]},$$

where β_0, \dots, β_N are the model's adjustable parameters calculated using the maximum likelihood method [13].

The logistic regression input is configured to receive the weak classifiers' outputs p_1, \dots, p_N , and the resulting indicator p is modeled at its output.

The proposed model is schematically shown in Fig. 1.

After constructing the ensemble (with training all regression models, all weak classifiers, and the decision rule), it is tested on the test sampling D_{tsf} data, and the behavior of the calculated abnormality indicator

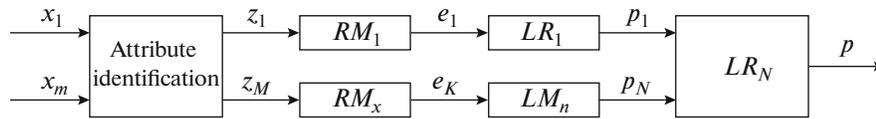


Fig. 1. Structure of the model for calculating the plant operation abnormality indicator. *RM* is a regression model and *LR* is a logistic regression.

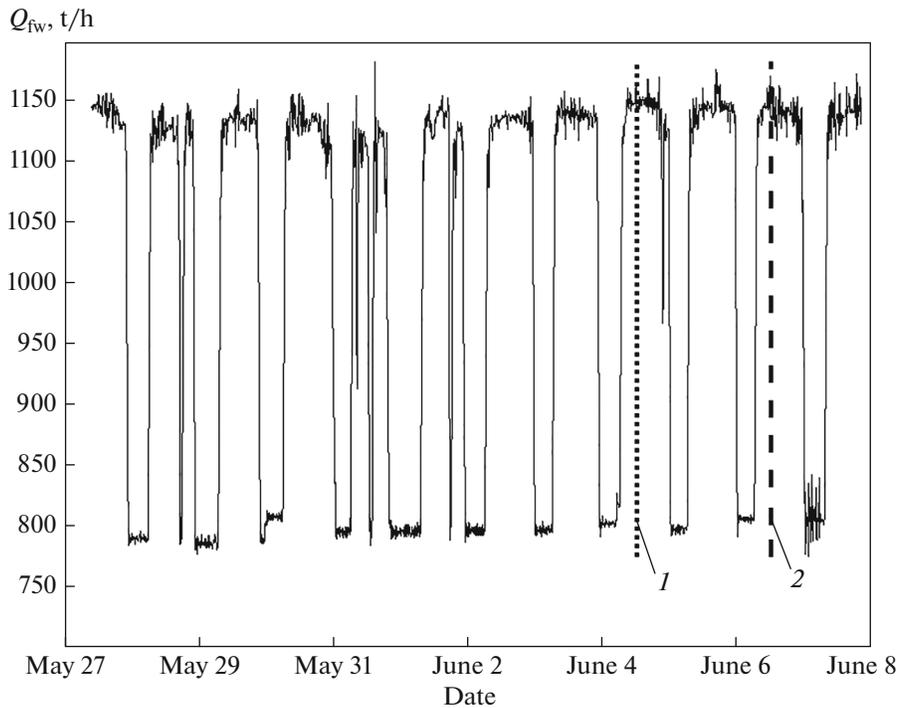


Fig. 2. Feedwater flowrate Q_{fw} versus the time interval before the emergency (condensate leak to the slip nut) (2014). 1—Start of the preemergency interval; and 2—emergency.

$p(t)$ in preemergency intervals of time is visually estimated.

EXPERIMENTAL INVESTIGATION RESULTS

Experimental investigations were carried out using the historical data on the operation of a 300 MW power unit’s feedwater pump recorded for 3 years. At each moment of time, $m = 45$ pump operation indicators (e.g., oil pressure, bearing vibration velocity, steam temperature, etc.) were recorded at $\Delta t = 5$ min intervals. After excluding the time moments at which the pump was in the disconnected state, the number of time readings made $T = 39\,629$. During the considered period of time, four defects in the pump operation were recorded: overheating of the driving turbine thrust pads, leak to the cover from the bearing side, and leak of condensate to the slip nut and to the end-face cover on the startup device side. For each malfunction, the moment of its recording is known (to an accuracy of 1 day). The class tags vector y was formed

in the following way: the values of $y(t)$ were taken equal to 1 for all moments of time starting from noon of the second day before recording the defect and ending with noon of the second day after it; all other tags were taken equal to 0. Figure 2 shows a fragment of the input data before one of the failures.

The set of all moments of time is subdivided into subsets T_{tr1} , T_{tr2} , and T_{test} in the ratio 40/40/20. That is, 40% of all available time readings randomly fall in the training samples T_{tr1} and T_{tr2} , and 20% of them fall in the test sample T_{test} .

Based on the observed pump indicators x_1, \dots, x_m , its derivative indicators z_1, \dots, z_M , $M = 89$ were calculated, which were used for constructing $K = 50$ regression models. The modeled variable and the composition of regressors were selected in each model in a random manner. If the regression model’s determination coefficient R^2 (which is interpreted as the extent to which the model corresponds to the data) on the training sample D_{tr1} was found to be less than 0.7, such a

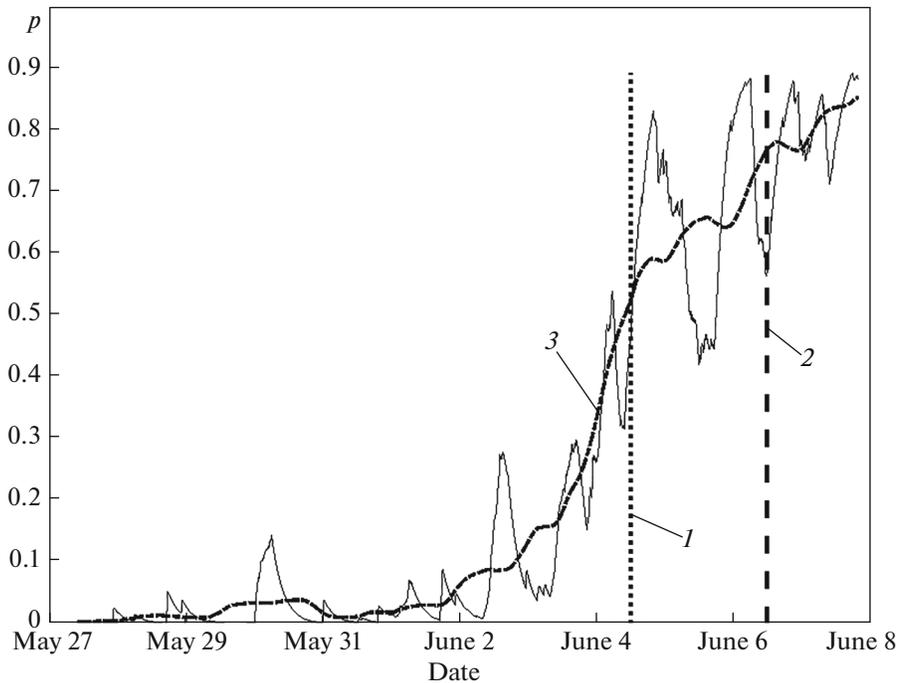


Fig. 3. Pump operation abnormality indicator versus time before emergency (2014). 1—Start of the preemergency interval; 2—emergency; and 3—“filtered”/smoothed abnormality indicator.

model was rejected. The average value of the determination coefficients R^2 of the constructed models is equal to 0.98 (the rms. deviation is equal to 0.04) on the training sample D_{tr1} and 0.97 (the rms deviation is equal to 0.07) on the sample D_{tr2} (which served as the test one for these models). It follows from the obtained values that the constructed regression models have a good generalizing ability on average; that is, the error probability on the test sample does not differ significantly from the error on the training sample.

For each regression model, the modeling errors $e_k(t)$, $k = \overline{1, K}$ were calculated at each moment of time t , $t \in T_{tr2}$, the values of which were used to construct $N = 20$ weak classifiers (logistic regressions). The number and composition of regressors for each logistic regression were selected in a random manner. If the quantitative interpretation of weak classifier quality (the AUC ROC indicator) on the training sample D_{tr1} was found to be less than 0.6, such classifier was rejected. The average value of the AUC ROC indicators of the constructed classifiers is equal to 0.95 (the rms deviation is 0.08) on the training sample D_{tr2} and 0.94 (the rms deviation is 0.11) on the test sample D_{tsr} . It follows from the obtained values that the constructed weak classifiers have a good generalizing ability on average.

The weak classifiers' outputs obtained on the training sample D_{tr2} were then used for training the logistic regression. The achieved value of the trained regression

model's indicator AUC ROC is equal to 0.99 on the training sample D_{tr2} and 0.98 on the test sample D_{tsr} .

Figure 3 shows the graphic dependence of the pump abnormality indicator p observed at the logistic regression model output during the preemergency interval of time.

As is seen from the graph, the abnormality indicator value begins to show a steady growth already on the morning of June 3, whereas the emergency interval in the training sample started only from noon on June 4. The early trend toward a growth of the indicator values testifies that the calculated indicator has a predictive ability.

The personnel operating the power unit equipment did not reveal any singularities in the behavior of individual pump performance indicators in the preemergency time interval (in particular, neither the emergency nor the preemergency state has any effect on the indicator shown in Fig. 2).

An alarm should be triggered when the abnormality indicator exceeds the preset limit ($p > p_{lim}$). The following notes should be mentioned in regard to this alarm:

(1) The value of p_{lim} must be established in the course of equipment pilot operation and must ensure almost guaranteed absence of false alarms.

(2) To decrease the probability of false alarms, the discrete signal $p > p_{lim}$ must trigger an alarm with a time delay of approximately a few hours in response to the leading edge or the abnormality indicator must be

smoothed by using filters (an inertial section or a moving average filter).

(3) The discrete signal $p > p_{\text{lim}}$ shall only be used to trigger an alarm. The decision to disconnect or shut-down the equipment must be made by the power plant personnel.

CONCLUSIONS

(1) An ensemble of regression models has been used for solving the MSET technique key problem concerned with selecting the composition of input and output variables used to construct a regression model. A logistic regression model has been applied for calculating the resulting abnormality indicator.

(2) A model featuring good generalizing and prediction abilities (AUC ROC ≈ 0.98) has been constructed as a result of carrying out experimental investigations of the proposed method using the actual data on operation of the PTN 1100-350-17-4 feedwater pump. The obtained results give grounds for using the proposed model as a basis for developing a system for monitoring the plant state and predicting future emergencies.

(3) The proposed method needs additional approbation on other archived samples, in off-line and online modes on current data, sophistication in regard to predicting the timeframes of reaching premergency and emergency technical state and determining particular defect causing factors.

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Translated by V. Filatov